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Mathematical Creativity: Student Geometrical Figure Apprehension in Geometry Problem-Solving Using New Auxiliary Elements

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Abstract: The definition of creativity among professional mathematicians and the definition of mathematical creativity in the classroom context are significantly different. The purpose of this study was to investigate the relationship between students' mathematical creativity (i.e., cognitive flexibility) and figure apprehension when solving geometric problems with novel auxiliary features such as straight lines and curved lines. In other words, this study determined if geometry knowledge influenced mathematical creativity (cognitive flexibility) in problem-solving. Grade-12 students participated in the intervention. The high school that is the research topic attempts to equip students with academic abilities and is, except for vocational schools, the most popular form of high school among all other types. Such a school was chosen for the study so that a significant proportion of students in Makassar could be represented. In this study, we discovered a relationship between cognitive flexibility and the geometric ability of pupils while solving problems involving auxiliary lines. This indicates that the usage of auxiliary lines as a reference for developing pupils' creative thinking skills must be advocated. In addition, good geometric abilities (e.g., visual thinking, geometrical reasoning) will encourage pupils to generate various problem-solving concepts. This finding contributes significantly to future research by focusing on auxiliary lines.

Keywords: *Cognitive flexibility, geometrical figure apprehension, mathematical creativity, new auxiliary elements.*

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Introduction

In recent years, developing unique and innovative solutions has become essential to address the difficulties of today's society. Creativity is an important attribute that everyone should possess, seeking to drive human progress and evolution (Leikin & Lev, 2013). Worldwide, governments and educational institutions have also recognized the relevance of creative thinking in pupils. Chen et al. (2015), Rahayuningsih et al. (2021), Hulsizer (2016) underline the importance of creative and flexible thinking in mathematics to enhance student capacities in all sectors and levels of education. From this perspective, creativity is an intrinsic part of mathematics (Brunkalla, 2009) and has been considered one of the primary components that must be promoted in mathematics education. In other words, mathematics is part of creative thinking (Singer et al., 2017).

As creativity is a comprehensive concept, this paper classifies the definition of creativity based on multiple perspectives. Mathematics divides creativity into four components: 1) creative stages or processes, (b) characteristics of creative actions and products, (c) creative individual personalities, and (d) cognitive processes underlying creative activities. Out of the various components of creativity listed above, the current study focused on the cognitive processes conducted in creative activities, namely solving geometric mathematical problems. Previous research has examined mathematical creativity in students' work in geometry. Previous research has examined mathematical creativity in students' work in geometry. The research carried out is based on two aspects: (a) examining the effect of geometrical figure apprehension on the production of multiple solutions and (b) how the need to construct auxiliary lines in a given shape drives the production of multiple solutions and creativity variables (Gridos et al., 2022). Gridos uses creative thinking indicators of fluency, flexibility, and originality. The analysis of the results shows that the way students

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perceive the geometrical figure and their ability to process it is an important factor in predicting their mathematical creativity. Furthermore, it became evident that only perceptual apprehension of geometrical figures is not a reliable predictor of creativity variables, as opposed to operative apprehension of geometrical figures that positively predict the characteristics of creativity: fluency, flexibility, and originality. Rahayuningsih et al. (2022) have developed indicators of creativity by looking at students' cognitive flexibility in solving problems. Rahayuningsih et al. argue that mathematical creativity in the school context is very different from professional mathematicians. Professional mathematicians view creativity in three aspects: fluency, flexibility, and originality. Therefore, in this study, researchers used cognitive flexibility indicators to measure student creativity. Specifically, this study aimed to explore students' mathematical creativity in solving geometric problems regarding geometrical figure apprehension in producing cognitive flexibility and using new auxiliary elements.

Literature Review

Creativity is a multifaceted phenomenon with varying definitions in various studies (Haylock, 1997; Leikin & Lev, 2013). Some definitions emphasize the nature of creative activities and products (e.g., Silver, 1997), while others focus on the stages of the creative process (Sriraman & Lee, 2011).

Torrance (1994) defines creativity from various aspects, such as fluency, flexibility, originality, and elaboration. However, creativity in mathematics refers to three aspects: fluency, flexibility, and originality. Fluency relates to ideas, flexibility refers to the ability to provide different ideas, and originality is related to the individual idea or product innovation (Leikin, 2009; Silver, 1997).

The definition of creativity among professional mathematicians and mathematical creativity in the classroom context is significantly different (Rahayuningsih et al., 2021; Singer et al., 2013; Sriraman, 2019) . In education, mathematical creativity refers to students' ability to contribute new insights or solutions to mathematical issues based on mathematical principles learnt in school, their prior experience in mathematical problem-solving, and the performance of student contributions (Rahayuningsih et al., 2021). Many prior scholars have devised measures for assessing student creativity from this perspective. Using multiple solution problem-solving is one of them. Multiple solution problem-solving tasks challenge students to explicitly answer math problems in multiple ways (Leikin, 2009). Leikin, (2009) contends that the various solutions to the issue are: (a) distinct interpretations of mathematical concepts; (b) alternative but equivalent definitions of mathematical concepts; and (c) diverse mathematical views and tools from other disciplines of mathematics. The structural evaluation of a completed solution or an attempt at a solution, with a view toward developing alternate solution approaches, is a crucial topic underpinning all the preceding (Mamona-Downs, 2008). Previous researchers have utilized multiple-solution problem-solving to measure and foster students' mathematical creativity, enhancing students' mathematical comprehension, cognitive flexibility, reasoning, and critical thinking (Elia et al., 2009; Levav-Waynberg & Leikin, 2012).

We sought a more suitable framework for researching and articulating the connection between students' mathematical creativity and their ability to tackle multiple mathematical problem-solving difficulties. We are interested in skills that enable students to manage their own learning, to assume, recognize, and address challenges that arise in unexpected circumstances in light of current knowledge trends. For this reason, we believe creativity studies might be more effectively targeted by analyses concentrating on key organizational theory concepts. According to Singer et al. (2013), the relationship between problem-posing and mathematical creativity can be evaluated within the context of organizational theory by analyzing students' cognitive flexibility. However, in this study, we used multiple-solution problem-solving tasks to assess students' mathematical creativity by examining their cognitive flexibility.

Cognitive flexibility is the capacity to adapt one's working methods to changing task requirements (Singer et al., 2013). Cognitive flexibility can be conceived via the lens of three primary constructs: cognitive variety, cognitive novelty, and change in cognitive framing (Furr, 2009; Spiro et al., 1992). Cognitive variety is the diversity of problem-solving thinking patterns within a group (Eisenhardt et al., 2010) or the diversity of cognitive patterns or views (Furr, 2009). Cognitive novelty relates to notions connected to content learning, students' overall content mastery (Orion & Hofstein, 1994), or the addition of an external perspective (Furr, 2009). Cognitive framing is a phenomenon that might result from the influence of external viewpoints, such as previous contextual experiences. Cognitive framing attempts to address new problems by employing previously employed solutions (Goncalo et al., 2010). Cognitive flexibility is a viable alternative for identifying the specific qualities of the creative abilities of mathematics high achievers for the reasons outlined above. In this study, mathematical creativity was examined by analyzing the behavior of high-achieving students as they developed various problem-solving strategies.

Among the various lessons in mathematics, geometry can be utilized to cultivate alternative modes of thought. Geometry offers study and proof opportunities comparable to the work of mathematicians (Singer et al., 2017). Multiple methods to a single problem can be seamlessly integrated via geometry. Practically any geometry issue in conventional textbooks may be transformed into a multiple-solution problem (Levav-Waynberg & Leikin, 2012). The following indicators are used to evaluate the mathematical creativity of pupils when solving geometric problems.

Table 1. Indicators of Mathematical Creative Thinking in Terms of Creative Flexibility

No	Indicators of mathematical creative thinking ability	Operational Definition
1	Cognitive novelty	• Find new strategies for solving a problem. • Display a new mindset
2	Cognitive variety	• Plan and use various resolution strategies when faced with complex problems and deadlocks. • Change the problem-solving strategy when faced with deadlocks. • Think of different ways to solve the problem. • Provide a variety of ways to solve the problem
3	Cognitive framing	• Take detailed steps to find a deeper meaning for the answer or solution to the problem

This study employed Duval, (1995, 2017) theory regarding students' geometry comprehension. Geometric drawings are required to analyse geometric problems because they facilitate an intuitive grasp of the relationship between visual components (Duval, 1995). According to Duval, many representations can be used to show mathematical errors. When trying to enhance the mathematics teaching and learning process in the classroom, it is necessary to consider two significant factors. First, the approach depends on activities that require several visual representations, such as drawings, curves, and tables. The second factor pertains to using computers for mathematical visualization (Presmeg, 2020) in geometry or calculus and geometry or graphics software that provides students with ample room for creative expression and visual inquiry.

Using Duval's perceptual approach to geometry, this study investigated the process of figure apprehension Duval (1995, 2017). In Duval's framework, the figure apprehension process consists of four types: perceptual, discursive, sequential, and operative. Each of these processes, according to Duval, serves to identify mathematical correlations in a seen image and works interactively during the problem-solving process. To appropriately represent this relationship, figure apprehension mechanisms must be designed to be distinct from one another (Duval, 1995).

Perceptual apprehension is the stage where students receive information about the structure of a geometric image (configuration). This perceptual apprehension process includes providing information about the name and area of a shape and recognizing the basic geometric components of a shape (points, line segments, triangles, circles, etc.). Perceptual knowledge also includes the identification of picture components. This sort of apprehension is static, and there is no discernible relationship between apprehension and visual structure (Duval, 1995). According to Elia et al. (2009) & Michael et al. (2009), perceptual apprehension is the initial exposure to the geometric form of a plane or space. Perceptual apprehension indicates the capacity to comprehend shapes and many subfigures inside a recognizable form.

Perceptual apprehension alone is insufficient to determine the mathematical features of a geometric image. Additionally, some initial information regarding an image is required. Establishing relationships between images and mathematical principles (definitions, theorems, axioms etc.) based on the initial information provided is called discursive apprehension (Duval, 1995, 2017; Michael-Chrysanthou & Gagatsis, 2014).

Using tools to draw a geometric figure enables pupils to gather knowledge about the shape and detect geometric relationships and solve a problem. Thus, according to Duval's Cognitive Model, the sequential apprehension process may be traced by using tools to construct geometric pictures. Moreover, focusing attention on several image components is known as operative apprehension. Operative apprehension entails modifying the shape of the initial image by adding lines, decomposing it into several components and rearranging it into several other images or modifying its position and orientation. From the definitions of each stage of the figure apprehension process, Table 2 reveals the indicators of the figure apprehension process on student behaviour.

Table 2. Indicators of Figure Apprehension

Perceptual Apprehension	Discursive Apprehension	Operative Apprehension	Sequential Apprehension
Can recognize images and the essential components of geometry and can name them	Can change the verbal information provided (information provided about objects in terms of symbolic representations and concepts) into visual information	Can describe the geometric images in the problem and rearrange them into different images by adding auxiliary lines in the form of straight lines or curved lines	Can create geometric shapes using tools

Table 2. Continued

Perceptual Apprehension	Discursive Apprehension	Operative Apprehension	Sequential Apprehension
	Accurately generate images when making inferences about geometric relationships	Can focus on certain parts of the image and can change the image by adding or removing new elements of geometry	Can determine how to draw geometrically using tools
	Can accurately convert visual information given in pictures into verbal information using symbols, notation and mathematical concepts and can draw correct conclusions	Can change the position or direction of specific images or their sub-elements by adding auxiliary lines in the form of straight lines or curved lines	

The above-described four forms of apprehensions can be reduced to two types of apprehensions (Duval, 2017). The perceptual method is the figure's spontaneous recognition. The mathematical method is associated with operative apprehension of the geometric figure. Thus, it involves regulating the figure's recognition based on its properties, from which other properties are retrieved. Introducing new auxiliary elements is one method for changing a given figure when addressing geometrical problems. According to Polya (2004), an additional element is a component that aids in the problem-solving process. Building new auxiliary elements, such as lines, is the most crucial aspect of geometric proof (Senk, 1985) and one of the four categories of problem-solving difficulty in geometry (Hsu & Silver, 2014). According to this explanation, there are two categories of auxiliary lines: straight lines, such as continuous thick lines, continuous thin lines, thick dash lines, and curved lines, such as ellipses, parabolas, and hyperbolas.

Research Objective

Based on the theoretical analysis presented above, it can be concluded that mathematical creativity and figure apprehension are concepts or themes that have been widely investigated in mathematics education and psychology separately. Unfortunately, very few qualitative studies relate these components to mathematics education research, such as tracing students' figure apprehension when producing new auxiliary parts in geometry problem-solving. Consequently, this study aimed to explore the relationship between students' mathematical creativity (cognitive flexibility) and figure apprehension when solving geometric problems with additional auxiliary elements, such as straight and curved lines.

Methodology

Research Design

The study group comprised 38 pupils in the twelfth grade (12 girls and 26 boys) aged 14-15. The average student is in the upper middle class of economics. Students have heterogeneous social statuses. The study was conducted at a public senior high school in Makassar that recruited its students based on a centralized national exam. It is a school with average student performance in its region. Public high schools in the city of Makassar have a diversity of religions, and social, cultural and economic statuses. All pupils in grade 12 at this high school participated in the intervention. The high school that is the research topic attempts to equip students with academic abilities and is, except for vocational schools, the most popular form of high school among all other types. The school was chosen for the study so that a significant proportion of students in Makassar could be represented. Within the geometry learning subdomain, these pupils studied triangles, polygons, geometric objects, and transformation geometry in secondary school. Before geometry classes were offered to pupils in grade 12, the research was conducted. In this manner, the study aims to expose the structure of students' figures of apprehension and mathematical creativity (cognitive flexibility) prior to enrolling in high school geometry classes.

Instrument and Codification

The instrument in this study was an open-ended geometry problem-solving test that had been developed and modified to measure mathematical creativity in terms of cognitive flexibility and figure apprehension indicators, named the Geometrical Figure Apprehension Creative test (GFACT). The test was developed by identifying indicators of cognitive flexibility (Rahayuningsih et al., 2022) and figure apprehension based on the Duval Cognitive Model (Duval, 2017), namely perceptual apprehension, operative apprehension of geometrical figures with a focus on reconfiguration and operative apprehension of geometrical figures with a focus on the introduction of auxiliary lines. Table 3 contains indicators for the Geometrical Figure Apprehension Creative test (GFACT).

Table 3. The Geometrical Figure Apprehension Creative Test (GFACT) Indicators

The content of the questions Geometrical Figure apprehension creative	Indicators sought	Geometrical Figure apprehension creative
In this question, students are given a geometric form and instructed to write its component pieces in various ways (cognitive flexibility). Consequently, this assignment aims to assess if students recognize the basic geometric shapes that comprise the presented shape and whether they are capable of creative problem-solving by offering multiple solutions.	Recognizes and identifies the provided figure and its fundamental geometric components.	Perceptual apprehension
• Students are given a two-dimensional shape with a specific area and asked to make another shape with the same area. To do this, they are expected to change the initial shape with several new shapes with the same area. This task aims to measure whether students can change the given image and to determine students' creative thinking skills in generating a new two-dimensional shape with the same area. • The area ratios of two flat shapes are provided to the students. The objectives of this task are to enable students to calculate the area of each two-dimensional shape using straight or curved lines and assess their creative problem-solving abilities using novel or uncommon approaches.	Can focus on some components of two-dimensional shapes and modify the image by adding new geometric elements or removing existing ones. Does not require quantitative data to make changes to a given geometric figure (for example, assigning a numerical value to the side lengths of the created figure). Can change the position or direction of a particular image or its sub-components in various ways. Can break down a given geometrical figure and recompose the components to generate a new figure by adding straight or curved auxiliary lines.	Operative apprehension

The Geometrical Figure Apprehension Creative Test (GFACT) was used to analyze individual differences in pupils' geometrical figure apprehension. Students were asked to undergo a 120-minute exam. It consists of six problems about three facets of geometrical figure apprehension. The first consists of perceptual apprehension and a single task. The second is operative apprehension of geometrical figures with a focus on reconfiguration. It consists of a single task that can be solved algorithmically or through reconfiguration of the figure. The third goal involves the operational apprehension of geometrical figures, emphasising the introduction of auxiliary lines.

The subsequent section provides task examples for every aspect of the Geometrical Figure Apprehension Creative test (GFACT)

Perceptual Apprehension Task

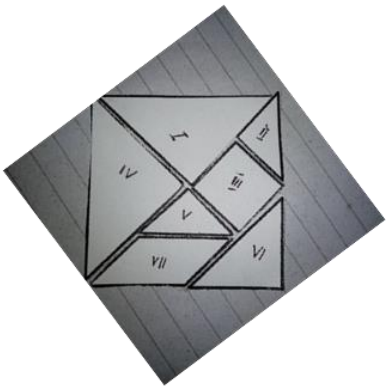

Figure 1. Perceptual Apprehension Task

See the arrangement of the tangrams that form the square above.

a. Explain the relationship between each tangram that forms the square. Explain how you worked on the task.

Operative apprehension task with the reconfiguration of the given shape and operative apprehension task with auxiliary line

b. Construct a shape with the same area as the rectangle.

c. Draw at least two forms with the same area as the rectangle.

d. Analyze one of the two-dimensional shapes in (b) and devise an alternate method for drawing the shape.

Analyzing of Data

To evaluate the test (GFACT) results, two mathematics professors developed a categorical scoring rubric (see Table 1 and 2). Using the markers of figure apprehension processes gleaned from a pilot study, a categorical scoring rubric was developed. Using the indicators, we determined the category with the highest score. This category was used as a benchmark to evaluate students' responses. The pilot study's student responses were then analyzed to determine the remaining categories with lower scores. In this grading system, the highest grade symbolizes the student's expected level of performance.

Consequently, the quality of the individual student's figure apprehension processes diminishes as the score decreases. Researchers independently rated student replies using a categorical scoring rubric developed beforehand. When there was a dispute in the scoring of student responses, those responses were re-evaluated to reach a unified scoring conclusion.

The percentage of pupils belonging to each category was determined, and based on these percentages, we attempted to discern students' figure apprehension processes. The procedure for analyzing each student's response data is as follows:

0: Unanswered or illogical answers (does not convert the given visual data to verbal data and draws inaccurate conclusions)

1: Accurately converts visual data to verbal data but draws incorrect conclusions

2: Accurately converts visual data to verbal data but draws no conclusions

3: Does not convert visual data to verbal data, but draws accurate conclusions

4: Accurately converts visual data to verbal data and draws accurate conclusions

After collecting the students' written responses, pre-interviews were done with volunteer students, and 12 students who could articulate their viewpoints explicitly were selected from various categories to participate in 20- to 25-minute clinical interviews. Clinical interviews were done so that data on students' cognitive processes about their thoughts and comprehension could be gathered and analyzed, and the reasoning behind their opinions could be uncovered (Clements et al., 2011). With the student's agreement, the interviews were recorded using a voice recorder. In these interviews, students were asked, "Could you explain and defend your response?" Then, the interviewer attempted to elucidate the rationales underlying the written responses. Based on the results of source triangulation, only two students succeeded in becoming research subjects according to the characteristics the researcher wanted.

The degree of trustworthiness of the data was increased by (a) ensuring that the data collected were rigorous and comprehensive, by managing assignments in written form and producing verbal transcriptions of each interview immediately after recording; (b) validating the coding and recoding processes of different categories through discussions with several mathematics education experts. A professor and two lecturers with doctoral degrees in mathematics instruction were asked to participate in the discussions. In the results section, we discuss the findings based on emergent themes, comparing and contrasting them with past studies' findings.

Findings / Results

The results of the GFACT analysis indicate that the majority of the participants' geometry knowledge falls into the category of *"accurately converts visual data to verbal data but draws incorrect conclusions".* However, two of them were able to solve the questions with good grades. These students' answers were categorized into *"accurately converts visual data to verbal data and draws accurate conclusions*". We interviewed the two students as research participants to investigate the relationship between mathematical creativity (cognitive flexibility) and figure apprehension in solving geometric problems.

The Overview of Subject 1 Geometrical Creative Figure Apprehension

When responding to the tangram arrangement that composes a square, Subject 1 referred to the image in the question. The subject appeared nimble and re-sketched the presented image, demonstrating that the student could identify new approaches to problem-solving (creative indicators appeared), and then elaborated on the relationship between each tangram. Subject 1 then explained that the sum of the areas of tangrams 1 and 4 equals the area of all other tangrams.

Subject 1 also defined the ratio of the area of each tangram and calculated a variety of areas for each square. This suggests that the student can prepare and implement a few problem-solving techniques when confronted with difficult problems and impasses. The following excerpt contains the research interview with Subject 1.

Interviewer: What do you know from the picture (while pointing to picture 1)?

Subject 1: I see a variety of different flat shapes.

Interviewer: What are the names of the shapes you see?

Subject 1: Small square, large triangle, medium triangle, small triangle, and parallelogram

Interviewer: What can you see from the shapes that make up the square?

Subject 1: Hmm. You mean the area?

Interviewer: Well, that for example, just describe it from your perspective! Anything interesting for you to tell? You are free to express it.

Subject 1: I see that tangram 1 and tangram 4 have the same area. The sum of the areas of tangrams 1 and 4 are the same as the sum of the areas of tangrams 2, 3, 5, 6 and 7. The area of tangram 6 is two times the area of the big triangles, which are tangrams 1 and 4. Tangram 2 and tangram 5 have the same area.

From the interview excerpt, it appears that Subject 1 could recognize and name each shape and its basic geometric elements. The student could focus on some components of the two-dimensional shape and could modify the shape by adding new geometric elements or removing existing geometric elements. In addition, the student could also change the shape into another shape with the same area. From the interview results, it appears that Subject 1 was able to create a two-dimensional shape with the same area as the seven tangrams. Subject 1 formed other two-dimensional shapes such as a trapezoid, a rectangle, and a parallelogram. The following interview excerpt shows the results of the study.

Interviewer: What shapes did you make?

Subject 1: a trapezoid, a rectangle, and a parallelogram.

Interviewer: How could you create those shapes?

Subject 1: I compiled and made the shapes on isometric paper so that the size of the shapes did not change.

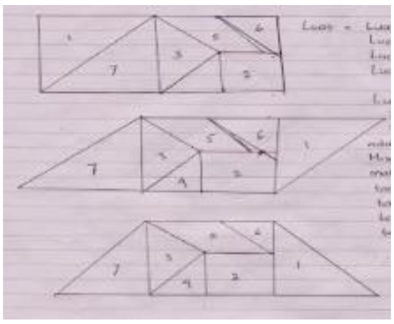

Figure 2. Subject 1's Response

From the interview excerpt above, it is obvious that Subject 1 could describe the existing geometric images in the problem and could rearrange existing components to create new geometric shapes by adding auxiliary lines in the form of straight lines on isometric paper. In addition, the student could convert visual data into verbal data by describing the broad definition of each two-dimensional shape. Although the area of the shape is not included in the question, the student could draw an accurate conclusion.

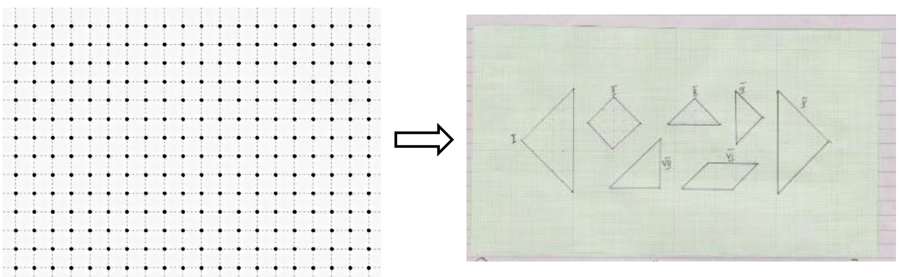

Figure 3. Student's Sketching on Isometric Paper

Interestingly, at the beginning of the task completion, the student focused on the text or narration of the question, rather than the pictures. The tendency was that the student looked for the instruction and narrative of the task instead of seeing pictures provided on the question sheet. The following interview excerpt supports this finding.

> *Interviewer: What were you doing here? (Pointing at the video recording and showing it to the student).*
>
> *Subject 1: I was looking for the test instructions.*
>
> *Interviewer: You couldn't do the task with only looking at the picture?*
>
> *Subject: I didn't understand the picture, and what the picture is for.*

The Overview of Subject 2 Geometrical Creative Figure Apprehension

Subject 2 began without significant movement or sound. The student appeared attentive and attempted to comprehend each image in the question and the instructions. However, the student could solve the task effectively. The results of the interview indicate that Subject 2 could recognize and identify the tangrams on the question and the fundamental components of geometry. Subject 2 could also accurately transform the visual information presented in the image into verbal information utilizing symbols, notations, and mathematical concepts to draw correct inferences. The following are excerpts from Subject 2's interview.

> *Interviewer: What were you thinking when you saw this?*
>
> *Subject 2: I will cut and separate all the existing two-dimensional shapes, these triangles look the same, don't they? These triangles are also the same. If I fold this shape into two (tangram 7), it turns out that this is the same as this triangle, hmmm.*
>
> *Interviewer: What are you thinking now?*
>
> *Subject 2: It looks like all shapes can be shaped into small triangles like this, Bu.*
>
> *Interviewer: Are you sure?*
>
> *Subject 2: Yes, I am.*

The interview extract demonstrated that the student could transfer visual knowledge into verbal information by confidently explaining significant concepts about two-dimensional forms. Additionally, the learner could describe the geometric images in the task and rearrange them by adding auxiliary lines to create new images.

> *Interviewer: How can you answer question no. 2?*
>
> *Student 2: I can make several different shapes from these two-dimensional shapes. However, do new shapes have to be squares, parallelograms, trapezoids? I think there are many other unique shapes that I can make, such as a fish, a train, or a small bus.*
>
> *Interviewer: Is this reasonable?*
>
> *Student 2: Yes, of course.*
>
> *Interviewer: How can you make it?*
>
> *Student 2: I can use the real picture of a fish and organize the shapes into it.*

The interview excerpt shows that the student used shapes in the tasks to form a new and unique picture. The student changed the image by adding new elements or removing existing elements in the geometric drawing, for example, by adding auxiliary lines such as curved lines and straight lines to create new two-dimensional shapes. The following is a new geometric drawing produced by Subject 2. In addition to a firm knowledge of geometry, the student displayed the capacity to generate unique ideas, adopt a novel perspective, think outside the box, and discern answers with plausible justifications.

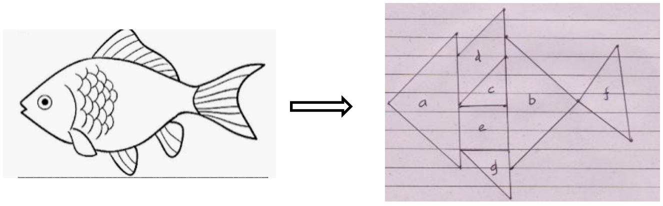

Figure 4. Student 2's sketching using curved lines

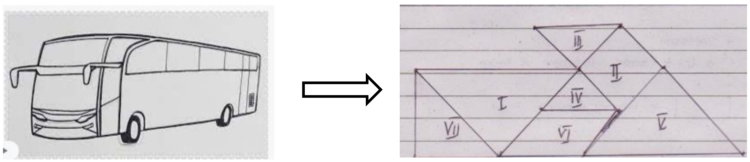

Figure 5. Student 2's sketching using curved lines

Discussion

The purpose of this study was to investigate the relationship between students' mathematical creativity (i.e., cognitive flexibility) and figure apprehension when solving geometric problems with novel auxiliary features such as straight lines and curved lines. In other words, this study determined if geometry knowledge influenced mathematical creativity (cognitive flexibility) in problem-solving. According to researchers, mathematical creativity should be fostered in all students (Sheffield, 2009). Creativity factors such as fluency and adaptability are dynamic and innately influenced by how individuals learn (Gridos et al., 2019; Leikin, 2009; Silver, 1997). The findings of this study demonstrate that comprehending geometry might influence students' creative thinking abilities, which in this case is cognitive flexibility. A solid grasp of geometry will enhance learners' capacity to think creatively. Duval, (1995) contends that "we might improve a student's fluency and flexibility with activities designed to overcome perceptual apprehension of the geometrical figure and lead to operative apprehension". The findings of this study also indicate that new answers or solutions to geometric issues can be produced with auxiliary lines, such as straight or curved lines.

The ability of students to show various ways or solutions requires more complex skills such as perception of the figure, not only in one form but also being able to see from several different forms. The contested capability is the mereological modification of the geometrical figure (Duval, 1999). Thus, students can simultaneously focus on multiple aspects of the provided form, recognize new elements, construct new elements in various forms, and develop diverse solution strategies (Gagatsis et al., 2015).

Auxiliary lines might stimulate students' creativity while tackling geometric problems. This study's results demonstrate that the *operative apprehension task with auxiliary line* refers to the relationship between the device used to produce an image and the image's mathematical qualities. It is assumed that students can develop accurate links between unit squares and mathematical principles based on the varied two dimensions provided.

However, the GFACT test revealed that more than half of the perceptual apprehension, operative apprehension tasks with the reconfiguration of the given shape, and operative apprehension tasks with auxiliary line did not achieve the top category score. Most students could not identify the various geometric shapes in the provided images; transfer verbal knowledge into visual information; generate verbal information based on visual information; draw inferences unaffected by the geometry of the image, or decipher and rearrange geometric forms. It demonstrates that secondary school students have a poor understanding of geometry and mathematical creativity. Students in junior high school should be able to prove theorems about two-dimensional geometry Tahmir et al. (2018). However, most pupils struggle even with fundamental tasks such as converting spoken information into visual form. According to Tahmir et al., the most important reason for this is the significance of raising the degree of knowledge in the learning environment. At the same time, students' cognitive processes and comprehension continue to be disregarded.

This study was also successful in indicating that students typically find it challenging to comprehend images without narrative. Many students in this study could not solve geometric problems without a narrative caption. Following the findings of Kim et al. (2022) study, most pupils are accustomed to reading text-oriented geometric description patterns. Kim et al. demonstrate further that most adult readers first check the text and then the relevant visual elements.

Conclusion

In this study, we discovered a relationship between cognitive flexibility and the geometric ability of pupils while solving problems involving auxiliary lines. This finding indicates that the usage of auxiliary lines as a reference for developing pupils' creative thinking skills must be advocated. In addition, good geometric abilities (e.g., visual thinking, geometrical reasoning) will encourage pupils to generate various problem-solving concepts. This finding contributes significantly to future research by focusing on auxiliary lines.

Recommendations

We also observed a tendency for pupils to encounter cognitive hurdles when interpreting difficulties presented as images. This problem can challenge pupils of all levels as they become accustomed to solving graphical problems. Students' unfamiliarity with these difficulties, as they are accustomed to handling procedural problems and rarely engage with problems with many representations, is undoubtedly the origin of this issue. In addition, we discovered that students tended to focus more on verbal or narrative problems than visual ones. For further research, the researcher hopes to reveal the involvement of students' creativity at the tertiary level, not only from geometric mathematical content but for some mathematical content such as real analysis, calculus and elementary algebra.

Limitations

Based on the findings of this study, we propose recommendations for creating computer software-based learning tools that can improve student creativity. The software-based tools can naturally reduce pupils' unfavourable assumptions while confronting image-based challenges. Future research should consider exploring the use of these tools in the learning process.

Authorship Contribution Statement

Muzaini: Conceptualization, first draft. Rahayuningsih: Editing, Final draft, data collection and analysis. Ikram: Data collection, data editing, data reduction. Nasiruddin: Data collection, data editing, data reduction

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